

Mach Cones in Viscous Matter

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Abstract. Employing a microscopic transport model we investigate the evolution of high energetic jets moving through a viscous medium. For the scenario of an unstoppable jet we observe a clearly strong collective behavior for a low dissipative system $\eta/s \approx 0.005$, leading to the observation of cone-like structures. Increasing the dissipation of the system to $\eta/s \approx 0.32$ the Mach Cone structure vanishes. Furthermore, we investigate jet-associated particle correlations. A double-peak structure, as observed in experimental data, is even for low-dissipative systems not supported, because of the large influence of the head shock.

1. Introduction

The elliptic flow coefficient v_2 measured in heavy-ion collisions at the Relativistic Heavy Ion Collider has a large value, leading to the indication that the created quark-gluon plasma (QGP) behaves like an almost perfect fluid [1, 2]. This is confirmed by recent calculations of viscous hydrodynamics [3] and microscopic transport calculations [4, 5]. Jet-quenching [6] has been discovered and very exciting jet-associated particle correlations have been observed [7]-[10]. They indicate the formation of shocks in form of Mach Cones induced by highly-energetic partons traversing the QGP [11]. Measurements of the Mach Cone angle could give us the possibility to extract the equation of state of the QGP.

In this work we address the question, under what conditions Mach Cones can develop in viscous gluon matter for given constant η/s values. Furthermore, we discuss a possible double-peak structure in jet-associated two-particle correlations.

2. The parton cascade BAMPS

The Boltzmann Approach of Multi Parton Scatterings (BAMPS) employed in this study is a microscopic transport model solving the Boltzmann equation

$$p^\mu \partial_\mu f(x, p) = C(x, p) \quad (1)$$

for on-shell particles with the collision integral $C(x, p)$. The algorithm for collisions is based on the stochastic interpretation of the transition rate [12, 13]. In this study, we consider only binary gluon scattering processes with an isotropic constant cross section, which is related to the η/s ratio via a simple relation [14, 15].

3. Shocks Waves and Mach Cones

Mach Cones, which are special phenomena of shock waves, have their origin in ideal hydrodynamics [16]. A very weak perturbation in a perfect fluid induces sound waves which propagate with the speed of sound $c_s = \sqrt{dp/de}$, where p is the pressure and e is the energy density. In the case where the perturbation with velocity v_{jet} moves faster than the sound waves, the sound waves lie on a cone with an emission angle $\alpha_w = \arccos(c_s/v_{\text{jet}})$. Considering a massless gluon gas, with $e = 3p$ and $c_s = 1/\sqrt{3}$, and a perturbation with $v_{\text{jet}} = 1$, the emission angle is

$$\alpha_w = 54,73^\circ. \quad (2)$$

In the case of stronger perturbations, the sound waves move faster than the speed of sound through the medium and therefore are called shock waves [17]. Then we can approximate the emission angle by

$$\alpha \approx \arccos \frac{v_{\text{shock}}}{v_{\text{jet}}}. \quad (3)$$

The velocity of the shock front v_{shock} depends on the pressure (energy density) in the shock front region p_0 (e_0) and in the stationary medium itself p_1 (e_1):

$$v_{\text{shock}} = \left[\frac{(p_1 - p_0)(e_0 + p_1)}{(e_1 - e_0)(e_1 + p_0)} \right]^{\frac{1}{2}} \quad (4)$$

Eq.(4) has the following limits: For $p_0 \gg p_1$ we obtain $v_{\text{shock}} = 1$. If $p_0 \approx p_1$, that is, a very weak perturbation, we get the expected limit of the speed of sound $v_{\text{shock}} \approx c_s$. In the latter case Eq.(3) reduces to the one given by α_w , as expected.

4. Transition from ideal to viscous Mach Cones in BAMPS

In this section we employ the microscopic transport model BAMPS to investigate Mach Cones with different strength of dissipations in the medium. All simulations are realized within a static and uniform medium of massless Boltzmann particles and $T = 400$ MeV. To save computational runtime we reduce our problem to two dimensions. The physical results compared to full three-dimensional simulations are similar except for the fact that the energy density decreases slower in our reduced geometry. Here we choose the xz -plane and apply a periodic boundary condition in y -direction.

A jet moving in positive z -direction is initialised at $t = 0$ fm/c at the position $z = -0.8$ fm. The jet is treated as a massless particle with zero spatial volume and zero transverse momentum, that is, $p_z = E_{\text{jet}} = 200$ GeV and $v_{\text{jet}} = 1$. The jet deposits energy and momentum to the medium via collisions with medium particles. In this scenario we neglect the deflection of the jet; its energy and momentum is set to its initial value after every collision.

In Fig.1 we demonstrate the transition from ideal Mach Cone to a highly viscous one by adjusting the shear viscosity over entropy density ratio in the medium from $\eta/s = 1/64\pi \approx 0.005$ to $1/\pi \approx 0.32$. The energy deposition of the jet is approximately $dE/dx = 11 - 14$ GeV/fm. The simulations are shown at $t = 2.5$ fm/c.

Using a non-physical small viscosity of $\eta/s = 1/64\pi$ we observe a strong collective behavior in form of a Mach Cone, as shown in the left panel of Fig.1. For comparison, the ideal Mach Cone caused by a very weak perturbation is given by a solid line. Its emission angle is given by (2). The shock wave, characterized by a higher energy density compared to the medium at rest, propagates with the emission angle α smaller than the ideal one, α_w . Due to the fact that the energy deposition is strong, the shock propagates through the medium faster than the speed of sound, given approximately by Eq. (3). A simulation where the energy deposition to the medium is much smaller reproduces the ideal Mach Cone with its emission angle α_w (2).

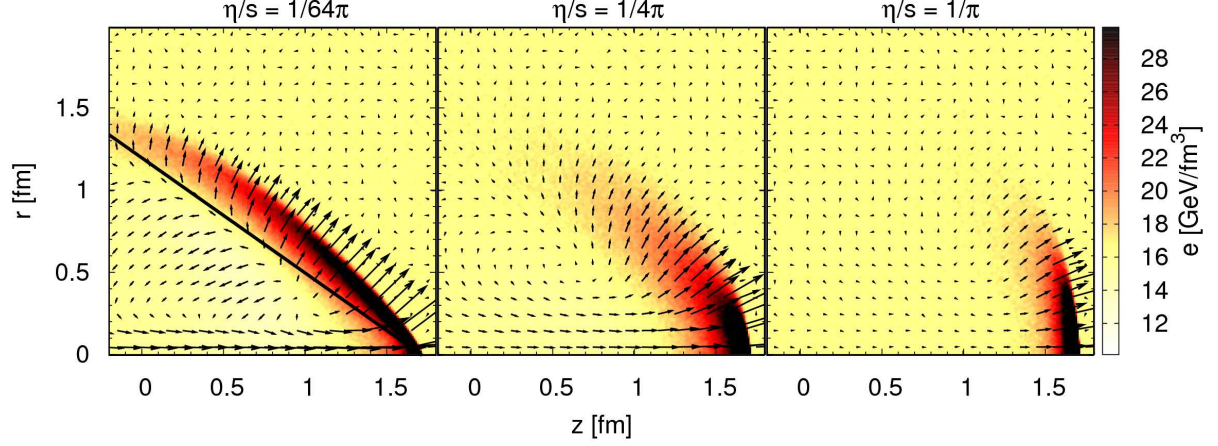


Figure 1. (Color online) The shape of a Mach Cone shown for different viscosities of the medium, $\eta/s = 1/64\pi$ (left), $\eta/s = 1/4\pi$ (middle), $\eta/s = 1/\pi$ (right). We show here the energy density plotted together with the velocity profile. Additionally, in the left panel the linear ideal Mach Cone for a very weak perturbation is shown by a solid line; its emission angle is (2).

Furthermore, a strong diffusion wake in direction of the jet, characterized by decreased energy density, and a head shock in the front are clearly visible. The results agree qualitatively with those found in [18]-[20].

If we increase the viscosity of the medium to larger values, shown in the middle and left panel of Fig.1, the typical Mach Cone structure smears out and vanishes completely. Due to stronger dissipation, the collective behavior gets weaker because of less particle interactions in the medium with a larger η/s . The results agree qualitatively with the analysis of one-dimensional shock waves realized in earlier studies using kinetic theory and viscous hydrodynamics [21]-[23], where a smearing-out of the shock profile is observed with higher viscosity.

5. Two-particle Correlation

The existence of a conical structure in Fig.1 for low viscosity are very exciting and naively lead to the assumption of a double-peak structure in jet-associated particle correlations. In Fig.2 we show the particle distribution $dN/(d\omega N)$ plotted vs. ω as extracted from BAMPS, where $\cos\omega = p_z/\sqrt{p_x^2 + p_z^2}$. We remark at this point that one has to mirror the results for $180^\circ - 360^\circ$.

In the left panel we show the particle distribution for all particles. A peak in direction of the jet at 180° is observed. A big contribution of the diffusion wake and head shock is present and hinders therefore the appearance of a double peak structure. In the same figure the particle distribution without the diffusion wake is plotted. However, the contribution of the head shock is still too large to get a double peak structure. A systematic cut in space is shown in the right panel, where we try to cut simultaneously a part of the diffusion wake and head shock. As one can see, a double peak structure appears and is enhanced with larger cut in space. However, such a cut in space does not relate to any experimental observation in heavy ion collisions and has to be treated with caution.

We conclude that a double peak structure exists, but the contribution of the diffusion wake and head shock might be stronger. This phenomena will be discussed in future studies.

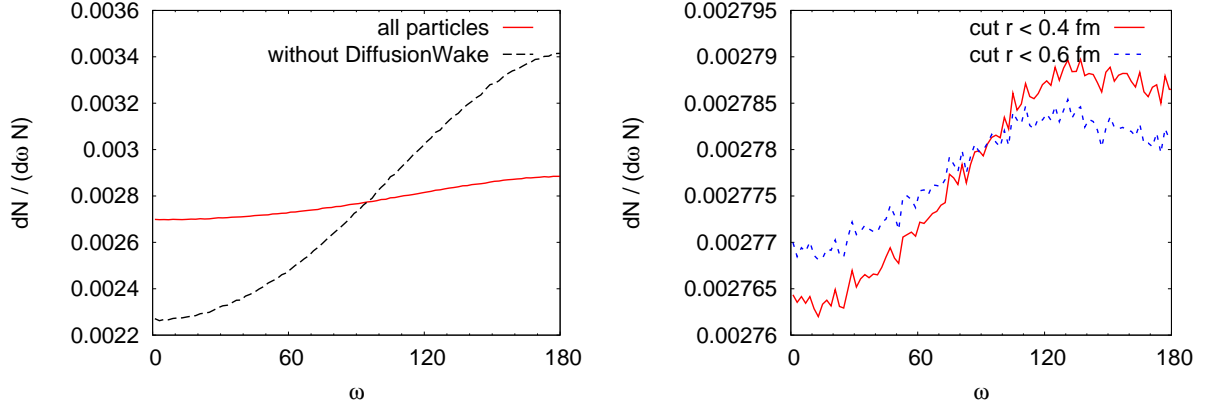


Figure 2. (Color online) Jet-associated particle correlations extracted from the simulation with $\eta/s = 1/64\pi$ as discussed in Fig.1. The left panel shows the correlations including all particles and one without the part of the diffusion wake. The right panel shows the correlations by systematically cuts in space to subtract the contribution of the diffusion wake and head shock. r is the transverse direction to the z -axis.

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